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In a recent paper by Professor G. W. Pierce[1] are described various effects which occur when a magnetic rod has superimposed upon it an alternating axial magnetic field of a frequency in the neighborhood of the natural mechanical frequency of the rod. Rods under the influence of magnetostriction can be made to play a part somewhat analogous to quartz crystals under the influence of piezoelectricity.

If a ferromagnetic rod or tube is clamped so that it lies along the axis of a coil, the combination becomes an electrical vibration instrument, and may be treated in the same way that telephone receivers are treated by Kennelly and Pierce,[2] or by Kennelly in his book on vibration instruments.[3] The present paper describes the measurement of the dynamic electrical characteristics of various magnetostrictive rods and tubes, also the magnitude of vibration and other measurements that could be made conveniently at the same time.

Method of Measurement.—The motional resistance and reactance are measured by connecting the coil containing the rod into an alternating current bridge, as in the reference[2] mentioned above. In this particular experiment, the bridge was designed to pass a direct current as well as the alternating current through the coil, thus it was possible to maintain a known field on the rod at any time. The coil is a long solenoid to insure a uniform magnetic field. The coil is 120 cm. long, has an inside diameter of two inches, and at its center developes a field of 32.5 gauss per ampere. The rods are all about 90 cm. long, and their natural frequencies lie between 2300 and 3000 cycles per second. In all measurements the superimposed alternating field is small compared to the direct or polarizing field.

In measurement of amplitude of vibration, an extremely light phosphor bronze strip upon which is mounted a small mirror is fastened from the end of the rod to the stationary frame to which the center of the rod is clamped. The longitudinal vibration of the rod thus gives an angular motion of the mirror. A point source of light reflected from the mirror is observed in a telescope with a

[1] G. W. Pierce—Magnetostriction Oscillators, These Proceedings, Vol. 63, No. 1, 1928.

[2] Kennelly and Pierce—Proc. Am. Acad., Vol. 48, No. 6, p. 113.

[3] Kennelly—Electrical Vibration Instruments.

micrometer eyepiece. By observing the motion of the spot of light in the telescope, and by measuring certain lengths (mirror arm etc.) the actual vibration amplitude can be calculated.

Effect of motion on resistance and reactance of solenoid with axial magnetostrictive rod.—Below are shown two curves typical of the electrical characteristics of the circuit near the resonance frequency of the rod.

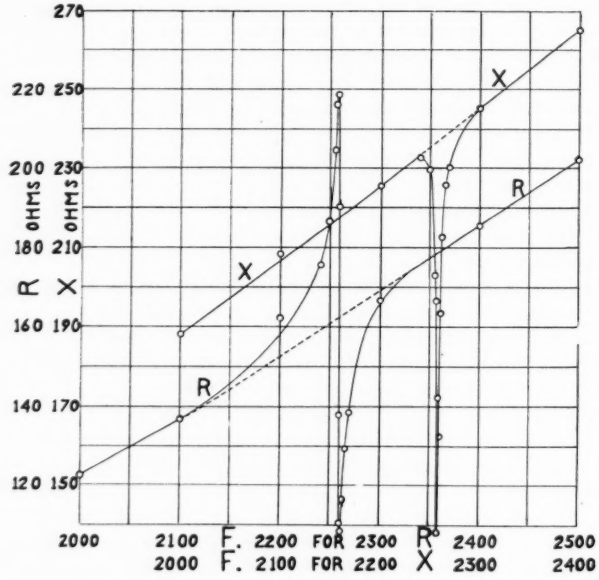

FIGURE 1.

Resistance R and reactance X plotted against frequency.
Stoic Metal Rod # 1 (length 92.43 cm. diameter .631 cm.).
The dotted part of the curves indicates damped values.
To avoid confusion, the frequency scales of the curves are displaced 100 cycles.

The large departure of R and X with the rod free (indicated by continuous curve) from the damped values (indicated by dotted curve) is due to the motion of the rod as the frequency of the oscilla-

tions approaches and passes the natural frequency of the longitudinal vibration of the rod. The values obtained by subtracting R and X of the damped system from the R and X of the free system have been called by Kennelly and Pierce motional values. The motional R (for the experimental conditions of the particular case shown above) ranges from -50 to $+50$ ohms approximately, and the motional reactance ranges from 0 to -100 ohms.

Results of measurements on rods.—Readings were taken on ten specimens in all. Three of these ten are of *stoic* metal (about the composition of invar steel), six are of commercial Grade A *nickel*, and one is of *stainless steel*. The different specimens of stoic and

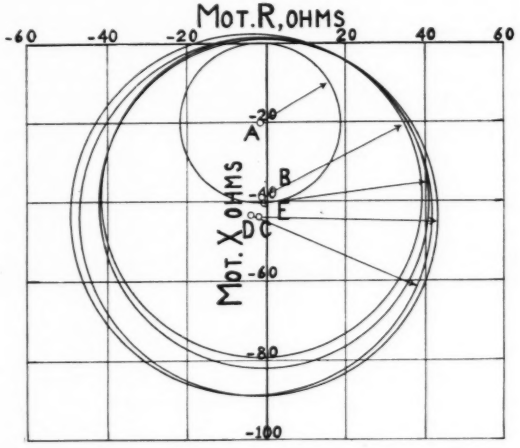

FIGURE 2.

Motional R plotted against motional X.
Stoic Metal Rod # 2 (length 94.38 cm. diameter .471 cm.).
Alternating field = .325 gauss.

Curve	*Polarizing field*	*Max. motional impedance*
	gauss	*ohms*
A	4.75	40.2
B	8.49	80.8
C	15.18	91.2
D	20.32	91.6
E	27.63	82.8

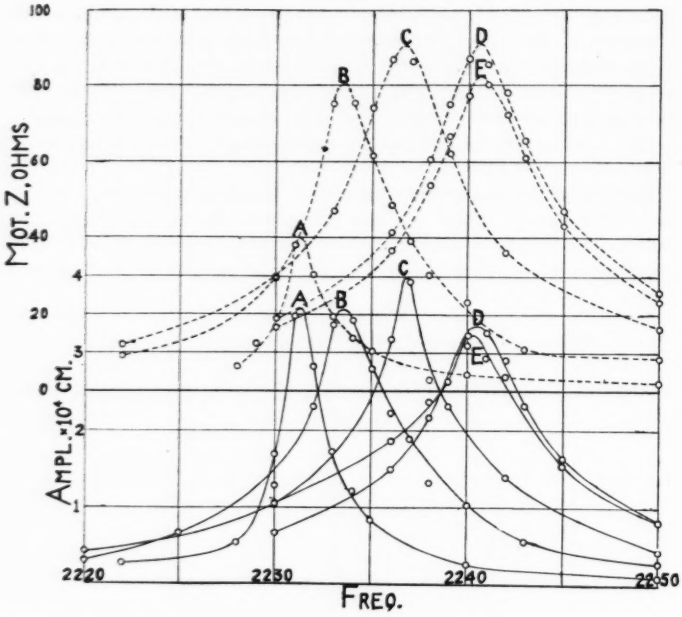

FIGURE 3.

Motional impedance plotted against frequency, dotted curves.
Amplitude plotted against frequency, smooth curves.
Stoic Metal Rod # 2.
Alternating and polarizing fields as in Figure 2.

of nickel are of different diameters, and it was hoped that the law of variation of motional impedance with diameter could be deduced, but owing to the fact that different specimens had different heat treatments, the results (other than that motional impedance increases with increasing diameter) are inconclusive.

One set of curves for each kind of metal is given. (See figures 2 to 7.) Each set has three types of curves.

Type 1.—Graphs of motional resistance against motional reactance. (*cf.* Kennelly and Pierce, also Pierce, references 2 and 1). These graphs are called circle diagrams.

Type 2.—Graphs of real motional impedance (motional Z) against frequency.

Type 3.—Graphs of amplitude of vibration of one end of the rod against frequency.

The failure of some circles to pass through the origin is due to a slight uncertainty in the determination of the damped values of R and X.

It can be seen at once that the motional impedance circles are very dependent on the polarizing field H. Curves showing the variation of maximum motional Z with H are shown later.

The motional impedance of rods is independent of small variations in the intensity of the alternating field. In measurements on tubes a dependence did appear.

A comparison was made between the amplitude of vibration and the motional impedance. In a majority of cases the motional impedance is found to be proportional to the amplitude. Variations

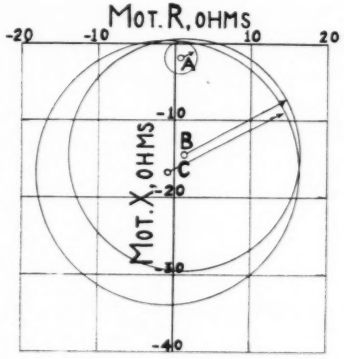

FIGURE 4.

Motional R plotted against motional X.
Nickel Rod # 6 (length 90.00 cm. diameter .790 cm.).
Alternating field = .325 gauss.

Curve	*Polarizing field*	*Max. motional impedance*
	gauss	*ohms*
A	14.7	4.4
B	21.1	30.6
C	27.7	34.4

from proportionality of from two to ten per cent did appear, but the first power law more nearly fits the facts than any other simple relation. For small values of amplitude and motional impedance the errors are too great to allow the relation to be checked, but for large values the proportionality of the two quantities seems fairly certain. To the limit of accuracy of the experiment the maximum amplitude and maximum motional impedance occur at the same frequency.

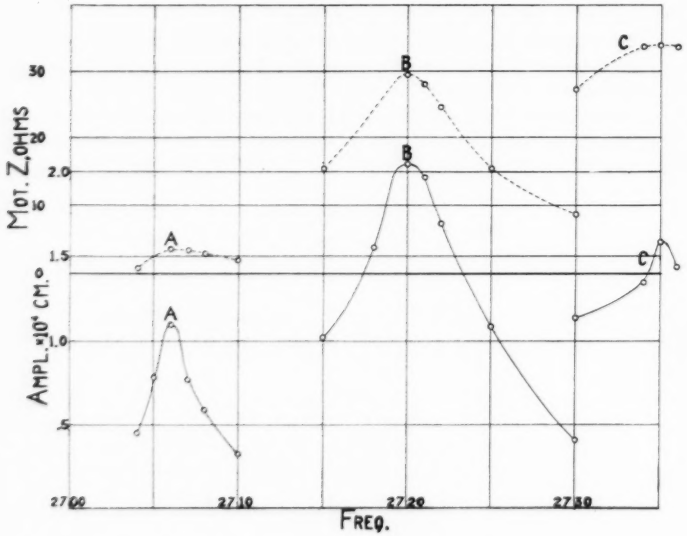

FIGURE 5.

Motional impedance plotted against frequency, dotted curves.
Amplitude plotted against frequency, smooth curves.
Nickel Rod # 6.
Alternating and polarizing fields as in Figure 4.

It is noticeable that the largest circle for stoic metal is about 90 ohms in diameter, for stainless steel about 70 ohms, and for nickel rod about 35 ohms. This gives a typical indication of the relative intensities of magnetostrictive reaction of these metals on any circuit. The reaction can be increased by suitable heat treatment.

Depression angle.—The depression angle is the angle between the diameter drawn from the origin and the positive motional resistance axis. As shown in a reference already mentioned,[2] this angle represents twice the lag of the magnetic flux behind the current in the coil. For all rods the depression angle is about 90 degrees, and for any one rod it stays essentially constant. This shows that for all rods studied the flux lags approximately 45 degrees behind the current

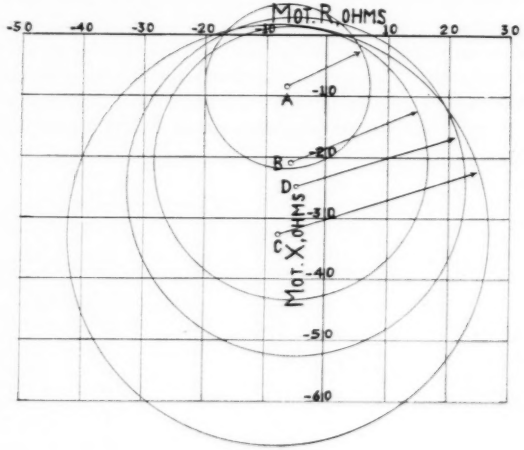

FIGURE 6.

Motional R plotted against motional X.
Stainless Steel Rod # 1 (length 90.00 cm. diameter .691 cm.).
Alternating field = .325 gauss.

Curve	*Polarizing field*	*Max. motional impedance*
	gauss	*ohms*
A	10.6	26.8
B	15.0	49.8
C	19.6	69.6
D	25.8	55.2

Decrement.—The decrement Δ of the system may also be calculated by methods described in reference[2]. The expression giving the decrement is

$$\Delta = 2\pi \frac{(f_1 - f_2)}{2}$$

where f_1 and f_2 are those frequencies for which the motional impedance is $\frac{1}{\sqrt{2}}$ times its maximum value. Owing to the extreme sharpness of the resonance curves accurate determinations of Δ are quite difficult. In all cases except that of stainless steel the decrement increases with increasing field. Stainless steel shows a decrease which may be regarded as abnormal. For the Stoic Metal Rod # 2 described above, the decrement per second varies from 10 to 30 as the field increases from 5 to 28 gauss. Nickel rods show decrements of the same order of magnitude. As the field on the stainless steel rod rises from 10 to 26 gauss, the decrement drops from 17 to 5. The logarithmic decrement per cycle (d) may be obtained by dividing Δ by f. The d for stoic metal changes from .00045 to .00134 as the

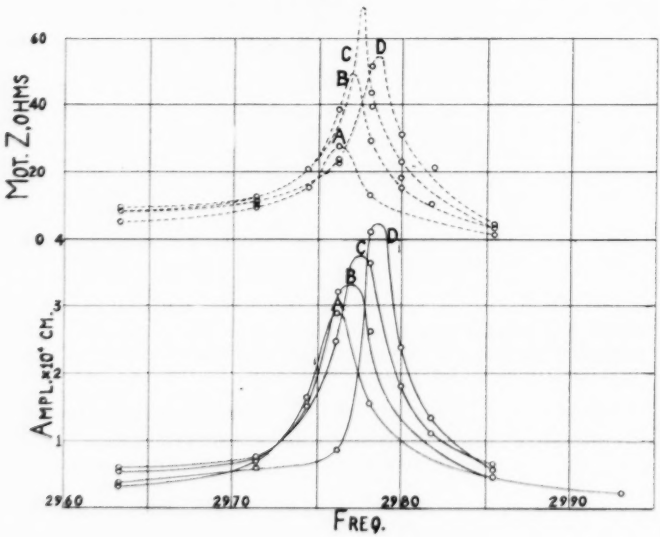

FIGURE 7.

Motional impedance plotted against frequency, dotted curves.
Amplitude plotted against frequency, smooth curves.
Stainless Steel Rod # 1.
Alternating and polarizing fields as in Figure 6.

field goes from 5 to 28, and for stainless steel the d changes from .00057 to .00017 as the field goes from 17 to 5. These decrements are very small compared to the decrements of ordinary electric circuits. If we should tune the coil used in these measurements to a frequency of 2350 cycles, the circuit would have a decrement per cycle of about 1.10.

Variation of motional impedance and amplitude of vibration with polarizing field.—Below are given sets of curves to show the maximum motional impedance and the amplitude of vibration at the resonance frequency of the rod as a function of H. In all cases the alternating field is the same, the field due to an alternating current of 10 milliamperes. One group shows the results for three stoic rods of different diameters, the other group of curves shows the results for the three rods whose circle diagrams are given above.

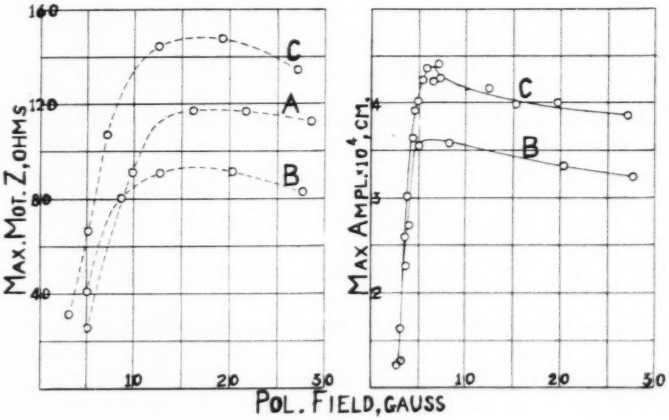

FIGURE 8.

Maximum motional impedance plotted against polarizing field, dotted curves.

Maximum amplitude plotted against polarizing field, smooth curves.

Alternating field = .325 gauss.

Curve	*Rod*	*Length cm.*	*Diameter cm.*
A	Stoic Metal # 1	92.43	.631
B	Stoic Metal # 2	94.38	.471
C	Stoic Metal # 3	86.60	.790

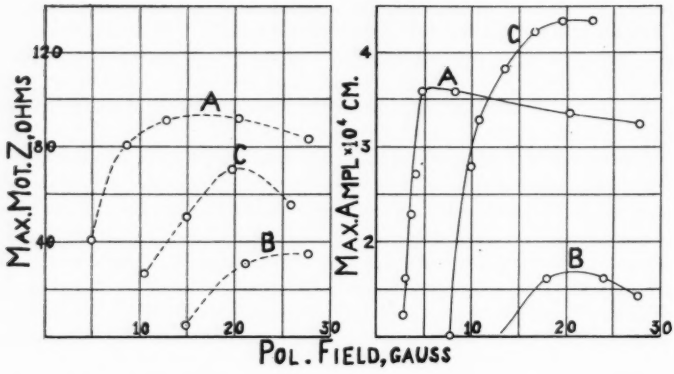

FIGURE 9.

Maximum motional impedance plotted against polarizing field, dotted curves.

Maximum amplitude plotted against polarizing field, smooth curves.

Alternating field = .325 gauss.

Curve	*Rod*	*Length cm.*	*Diameter cm.*
A	Stoic Metal # 2	94.38	.471
B	Nickel # 6	90.00	.790
C	Stainless Steel # 1	90.00	.691

Comparison with effects of static magnetostriction.—We can compare the effects of alternating and static magnetostriction in the following way. From the static magnetostriction curves, not shown here, we can tell what the change in length of a specimen would be if we increased the polarizing field from H to $H + \Delta H$. If we polarize the specimen at a field H and superimpose upon it an alternating field of the same frequency as the natural frequency of the specimen, and of intensity ΔH, we can measure the amplitude of vibration by the method I have already described. A comparison of the static increment in length with the amplitude shows the relative magnitude of the mechanical reactions. Such a comparison is given below.

TABLE No. 1

H gauss	static Δl for $H = .325$	dynamic Δl for $H = .325$	$\frac{\Delta l \text{ dynamic}}{\Delta l \text{ static}}$
	STOIC METAL ROD # 3		
2.59	$.0325 \times 10^{-4}$	1.22×10^{-4}	37.5
3.89	$.0298 \times 10^{-4}$	3.92×10^{-4}	131.5
5.70	$.0278 \times 10^{-4}$	4.42×10^{-4}	159.0
10.62	$.0271 \times 10^{-4}$	4.15×10^{-4}	153.0
17.09	$.0234 \times 10^{-4}$	4.01×10^{-4}	171.0
24.71	$.0177 \times 10^{-4}$	3.88×10^{-4}	219.0
	STAINLESS STEEL ROD # 1		
10.0	$.0739 \times 10^{-4}$	3.10×10^{-4}	41.9
15.0	$.0739 \times 10^{-4}$	4.10×10^{-4}	55.5
20.0	$.0739 \times 10^{-4}$	4.34×10^{-4}	58.5

It can be seen that the dynamic effect is from about 30 to 200 times as great as the static.

Variation of the natural frequency as a function of the polarizing field.—A knowledge of the rules governing the changes in the natural frequencies of magnetostrictive rods is of great importance, since the natural frequency can be used in the calculation of the velocity of sound in the metal, the elastic coefficient, etc.

It is found that the natural frequency is a slight function of the applied polarizing field in most cases. This effect is studied by G. W. Pierce[1] at higher frequencies.

The curves of figure 10 show the natural frequency vs. H (of the polarizing field) for the three specimens whose motional impedance circles are given above.

The dotted lines in each case represent natural frequencies determined when the polarizing field is cut off, and the rod is left with only its residual magnetism.

Despite the fact that stoic metal and stainless steel expand and nickel contracts for polarizing fields up to 30 gauss, the natural period increases with increasing H in every case. This was true for every specimen studied, no maxima were observed.

Split nickel tubes.—Since it appeared possible that the electromagnetic field did not penetrate deeply into the rods, it was logical to try some experiments on tubes, in which the field may penetrate a greater percentage of the volume. The tube used was split longitudinally to reduce eddy currents.

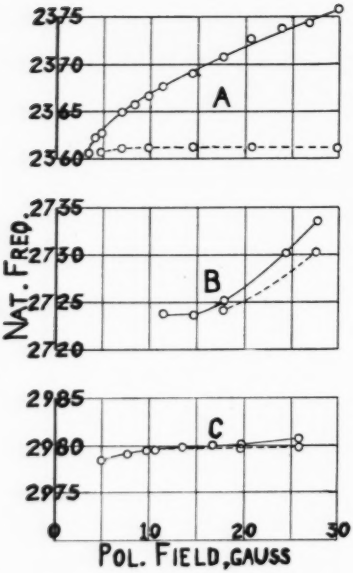

FIGURE 10.

Natural frequency plotted against polarizing field.
Dotted curves represent cases of residual magnetism.
Alternating field = .325 gauss.

Curve	*Rod*
A	Stoic Metal # 1
B	Nickel # 6
C	Stainless Steel # 1

In figures 11 and 12 are shown a set of curves for this nickel tube. Amplitudes of vibration were not measured in this particular case.

Several noteworthy facts can be seen at once. First, the depression angle here, unlike that for rods, is small. Second, the size of the circles is much greater than in the case of rods, especially nickel rods. Third, the decrements are larger in a tube than in a rod.

The effect of the variation of the intensity of the alternating field is illustrated in figure 13.

The four circles were obtained for values of alternating current ranging from .0079 to .0253 amperes. It can be seen that the circle diagrams are almost identical. The change in the diameter of the circles is from 201 to 191. This is a 5% change in motional impedance for a 320% change in alternating field intensity. As I have already mentioned, with rods the change in motional impedance for a similar change of intensity of alternating field was inappreciable.

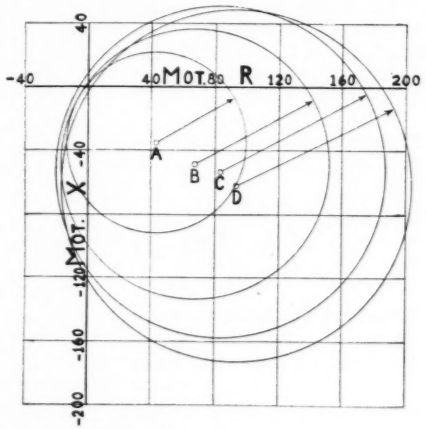

FIGURE 11.

Motional R plotted against motional X.

Nickel Tube # 1 (length, 90.00 cm., outer diameter 1.27 cm., thickness .064 cm.).

Alternating field = .325 gauss.

Curve	*Polarizing field*	*Max. motional impedance*
	gauss	*ohms*
A	9.3	112.6
B	13.9	169.8
C	18.9	208.0
D	23.3	220.0

Nickel tubes as sound sources.—Both the rods and tubes are strong sound sources when used as oscillators, but of the two the tubes are the more powerful. The sound energy comes from part of the motional electric power dissipated in the circuit. One noticeable

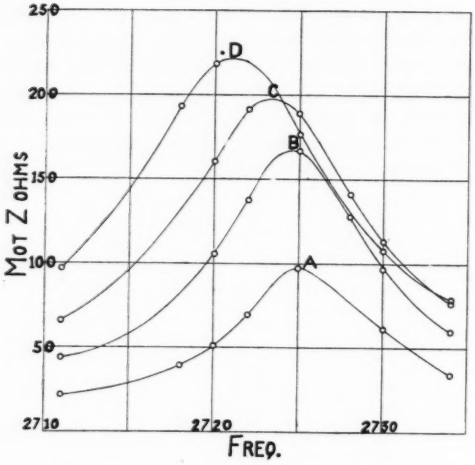

FIGURE 12.

Motional impedance plotted against frequency.
Nickel Tube # 1.
Alternating and polarizing fields as in Figure 11.

difference between rods and tubes is that at resonance considerable motional power is dissipated in the tubes, while rods at resonance have practically no motional resistance.

In order to increase the sound output, discs or cans may be fastened to the ends of the tube. Also much more electrical reaction may be obtained if the coils are compact ones about the center part of the tube. I did not use such a system for the preceding measurements because with short coils it is impossible to maintain a uniform field on the specimen. Measurements were made, however, on a device used by G. W. Pierce as a powerful sound source, consisting of a split nickel tube with open aluminum cans attached to the ends of the tube and a driving coil about its center. A sketch of this device is shown below with the circle diagrams.

The results are given in terms of direct current and r. m. s. alternating current flowing in the coil.

These results agree in character with those for the first nickel tube. The magnitude of the motional impedances is, of course,

much greater. The damped value of resistance is about 300 ohms, while the maximum motional resistance is about 1800 ohms. It is also to be noted that one particular polarizing current gives a maximum resonance motional impedance.

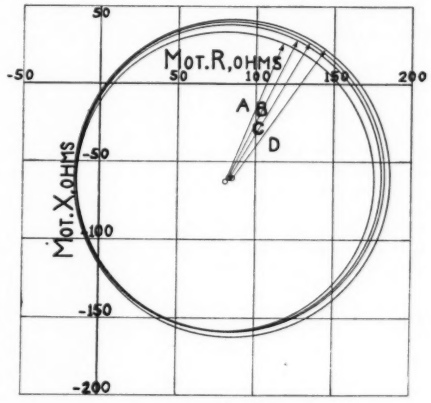

FIGURE 13.

Motional R plotted against motional X.
Nickel Tube # 1.
Polarizing field = 26.5 gauss.

Curve	*Alternating current in coil amperes*	*Max. motional impedance ohms*
A	.0253	191.0
B	.0178	193.6
C	.0136	196.8
D	.0079	201.0

The motional impedance of the system increases when the cans are removed, though the amount of sound radiated decreases. This is to be expected, it is analogous to the increase in motional impedance of a telephone receiver as the air pressure is reduced, an experiment described by Kennelly[3]. The actual motion of the tube is increased when the cans are removed, and therefore the electrical reaction increases.

Removal of the cans decreases the decrement.

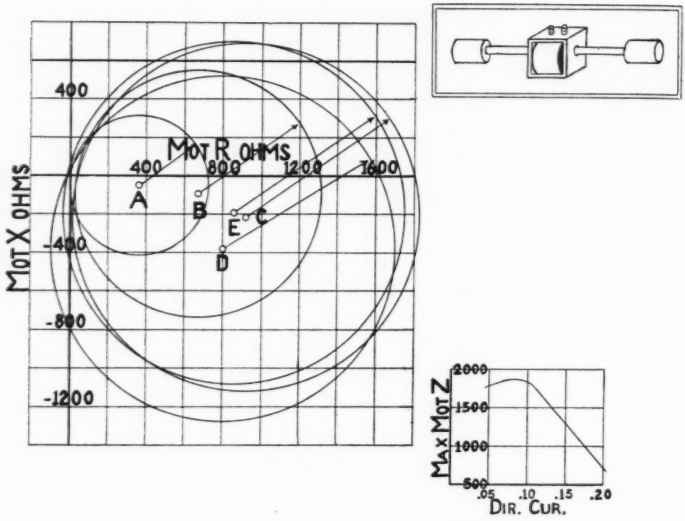

FIGURE 14.

Motional R plotted against motional X.
Maximum motional impedance plotted against direct current in the coil.
Nickel Tube # 2.
Inset shows arrangement of coil, tube, and cans.
Alternating current in the coil = .0070 amperes throughout.

Curve	*Direct current in coil amperes*	*Max. motional impedance ohms*
A	.200	730
B	.151	1274
C	.111	1836
D	.050	1790
	Curve E taken with cans removed.	
E	.151	1790

Damped R and X at 3300 cycles.
R 297 ohms X 4550 ohms
Resistance and reactance of the coil without the nickel tube.
R 200 ohms X 3300 ohms

Summary.—There follows an outline of the various results obtained.

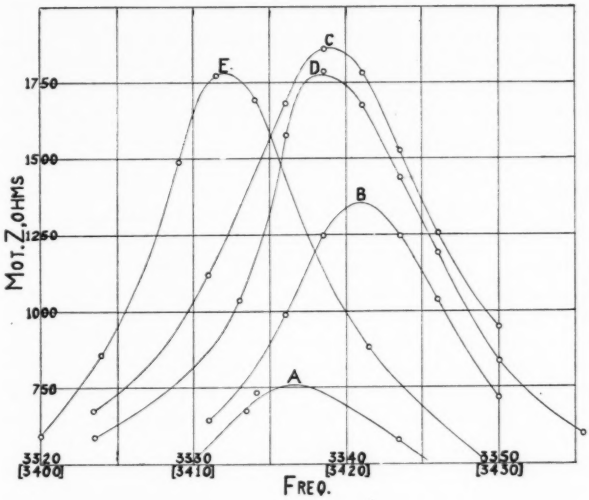

FIGURE 15.

Motional impedance plotted against frequency.
Nickel Tube # 2.
Alternating and direct current values as in Figure 14.
Frequency scale in brackets applies to curve E.

1. Alternating magnetic fields of a frequency equal to the natural mechanical frequency of a ferro-magnetic rod may give rise to displacements which are relatively several hundred times the displacements that occur with a stationary field.

2. The amplitude of vibration and all motional effects are found to be functions of the intensity of magnetization, or if one always works with a specimen initially demagnetized, the phenomena are functions of the applied polarizing field.

3. Motional resistance when plotted against motional reactance gives a circle of the same general nature as the circle diagrams that are obtained in work on telephone receivers. From these circles much information can be drawn.

4. The amplitude of vibration and the motional impedance are related to one another linearly to a reasonable approximation.

5. A study of the effect on the motional impedance of variation of

the polarizing field was made. Plotting maximum motional impedance against the polarizing field shows that in the case of stoic metal and stainless steel these curves have maxima; in the case of nickel these curves have no maxima up to a field of 30 gauss. This indicates that the electrical reaction of the system reaches a maximum for a low polarizing field in the case of the first two metals named; while for nickel the reaction increases to the highest fields used in this experiment.

6. A study of the effect on the motional impedance of variation of the alternating field shows that to a reasonable approximation the motional impedance is independent of alternating field; actually, for small alternating fields the motional impedance shows a slight linear decrease with increasing field.

7. The resonance amplitude of vibration is plotted against polarizing field, and it is found to reach a maximum for all cases studied at a field strength of less than 30 gauss. For small alternating fields, the amplitude of deflection is proportional to the field.

8. The angle of lag of the flux in the rod behind the current in the solenoid is found to be not far from 45 degrees for all the rods used. In most cases the lag angle appeared to be independent of polarizing or alternating field strength. The lag angle in tubes is much less.

9. From the circle diagrams the decrements of the system can be calculated. In all cases decrement increases with increasing polarizing field except in the case of stainless steel where there is a decrease.

10. In most cases the natural frequency of vibration of magnetostrictive rods increases with increasing polarizing field. In some cases the natural frequency is practically independent of the field. In no case was a decrease of frequency observed with increasing field.

11. Measurements on magnetostrictive sound sources show that a large motional power at resonance is obtainable, also that addition of sound radiating members, although increasing the amount of radiant sound energy, decreases the motional impedance and increases the decrement.

CRUFT LABORATORY, HARVARD UNIVERSITY
CAMBRIDGE, MASSACHUSETTS, U. S. A.